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Restriction to Parametric Resonant Decay after Inflation

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Abstract

We study parametric resonant decay of inflaton field with emphasis on its physical meaning. We show that the parametric resonance is indeed an induced process, which means that the more numbers of produced particles, the more inflaton field decays. We also consider the dissipative effects of produced particles and find that the dissipation reduces the resonant decay rate of inflaton field.

1 Introduction

Inflation [1, 2] is the most successful theory to solve many problems of the standard "hot big bang" Universe. In the inflationary Universe scenario the universe is expanded exponentially by the vacuum energy of some scalar field ϕ (inflaton). For this scenario to work, the radiation-dominated Universe must be recovered after inflationary stage. Therefore, some process is necessary to transfer the vacuum energy to relativistic particles. This process is called reheating.

The old version of reheating theory was first considered in [3, 4] for the new inflationary theory [5], and it can be applied to the chaotic inflation [6]. After inflation, inflaton field oscillates near the bottom of its effective potential. Inflaton field decays into other (lighter) particles due to its coupling to others. The reheating temperature can be estimated as $T_{RH} \simeq 10^{-1}\sqrt{\Gamma_{tot} M_p}$ [7], based on the single-particle decay. However, recent investigation revealed that the drastic decay of inflaton field ϕ occurs in the first stage of reheating [8, 9, 10, 11, 12, 13]. This stage is called preheating [8]. At first, inflaton field ϕ explosively decays into some bosons χ, whose spectrum is far from equilibrium. After χ-particles collide with each others or decay into lighter particles, thermal equilibrium can be achieved and reheating is complete.

The equation for the particles produced can be described by Mathieu type equation, which has instability solution in some regions of parameters (instability bands). If the relevant parameters stay in the instability bands long enough, the solution will explosively grow, which means that the number of particles created becomes exponentially large so that the parametric resonant decay takes place very efficiently [8, 9, 10, 11, 12].

From the different viewpoint, the physical meaning of parametric resonant decay could be considered as an "induced" decay, which is similar to an induced emission of photons in the two energy level system of atoms, in the sense that the presence of produced particles stimulates inflaton field to decay into those particles. Thus this phenomenon is peculiar to those particles that obey Bose-Einstein statistics, bosons, the number of which in each mode k can increase exponentially. This is the reason that parametric resonant decay into fermions cannot occur due to Pauli's exclusion principle [14, 8, 9, 10].

If the parametric resonance is an induced process, the parametric resonant decay

of inflaton into bosons are likely to be suppressed by the processes of scattering or decay of the produced particles [14, 9]. The reason is that dissipation (the particle scattering, decay and annihilation) will tend to decrease the mean occupation numbers N_k in each particular mode so that the induced decay will be reduced, and, as a result, these numbers cannot grow so large during the resonance period.

Therefore, in the present paper we study the parametric resonant decay in the narrow resonance region [1] with emphasis on its physical meaning and consider the effect of dissipation. In section 2, we will make clear that the parametric resonant decay is an induced effect, and we will see the restriction to that decay due to effects of dissipation in section 3. Section 4 contains our conclusion and discussion.

2 Parametric Resonant Decay as an Induced Effect

In this paper we assume the interaction between inflaton field ϕ and χ-particles as $\mathcal{L}_{int} = -\sigma\phi\chi^2$ and the effective potential of inflaton field as $V(\phi) = \frac{1}{2}m_\phi^2\phi^2$. The similar results can be obtained for other interactions and also for $\frac{\lambda}{4}\phi^4$ potential. The mode equation for χ field becomes

$$\ddot{\chi}_k + [\omega_k^2 + 2\sigma\phi_0 \sin(m_\phi t)]\chi_k = 0, \tag{1}$$

where $\omega_k^2 = k^2 + m_\chi^2$ with m_χ being the mass of the χ-particle, ϕ_0 is the amplitude of the inflaton field, and the expansion of the Universe is neglected.

We can expand χ field as

$$\chi = \frac{1}{(2\pi)^{3/2}} \int d^3k[a(k)\chi_-(t)\,\mathrm{e}^{i\vec{k}\vec{x}} + a^\dagger(k)\chi_+(t)\,\mathrm{e}^{-i\vec{k}\vec{x}}], \tag{2}$$

where $a^\dagger(k)$ and $a(k)$ are creation and annihilation operators, respectively, with commutation relation: $[a(k), a^\dagger(k')] = \delta(\vec{k} - \vec{k'})$. $\chi_\pm$ are the positive and negative frequency solutions of Eq.(1), which are equal to the positive and negative frequency asymptotic solutions ($= \exp(\pm k_0 t)/\sqrt{2k_0}$) if χ was free field.

Using Eq. (2) the energy density of the χ-particle can be calculated as

$$\begin{aligned} \langle 0\,|\,\rho_\chi\,|\,0\rangle &= \langle 0\,|\,\frac{1}{2}\dot{\chi}^2 + \frac{1}{2}\Omega_k^2\chi^2\,|\,0\rangle \\ &= \frac{1}{(2\pi)^3}\int d^3k \left[\frac{1}{2}|\dot{\chi}_-|^2 + \frac{1}{2}\Omega_k^2|\chi_-|^2\right], \end{aligned} \tag{3}$$

[1] We study onlyin the narrow resonance region since we expect that the suppresion will be unimportant in the broad resonance region.

where we take the vacuum expectation value. Thus the energy density for the χ-particle with momentum k is given by

$$\rho_\chi^k = \frac{1}{2}|\dot{\chi}_-|^2 + \frac{1}{2}\Omega_k^2|\chi_-|^2. \tag{4}$$

Then the time evolution of ρ_χ^k is described as

$$\dot{\rho}_\chi^k = \frac{d}{dt}\left(\frac{1}{2}|\dot{\chi}_-|^2 + \frac{1}{2}\Omega_k^2|\chi_-|^2\right) = \frac{1}{2}\frac{d(\Omega_k^2)}{dt}|\chi_-|^2, \tag{5}$$

where the Eq. (1) is used in the second step.

Hereafter, we assume that $\sigma\phi_0 \ll m_\phi^2$ which corresponds to so-called narrow resonance region. This assumption allows us to take perturbative analysis. [2] First, we will show that our analysis is general so that it does not depent on the exponential growth nature of the solution to (1). If we write the solution as

$$\chi_- = A\cos(\omega_k t), \tag{6}$$

where A is a slowly time dependent, complex function, the energy density can be given as follows:

$$\begin{aligned} \rho_k &= \frac{1}{2}|\dot{\chi}_k|^2 + \frac{1}{2}|\chi_-|^2, \\ &= \frac{1}{2}|A|^2\omega_k^2\cos^2(\omega_k t) + \frac{1}{2}|A|^2\omega_k^2\sin^2(\omega_k t), \\ &= \frac{1}{2}|A|^2\omega_k^2. \end{aligned} \tag{7}$$

From the equation (5), we get

$$\begin{aligned} \dot{\rho}_k &= \frac{1}{2}2\sigma\phi_0 m_\phi\cos(m_\phi t)|A|^2\cos^2(\omega_k t), \\ &= \frac{1}{2}\sigma\phi_0 m_\phi|A|^2\cos(m_\phi t)[1+\cos(2\omega_k t)], \end{aligned} \tag{8}$$

and taking the average over one period of oscillations of inflaton field ϕ, the rhs of this equation will not vanish when the resonance condition $\omega_k = m_\phi/2$ holds so that,

$$\begin{aligned} \dot{\rho}_k &\simeq \frac{1}{4}\sigma\phi_0 m_\phi|A|^2, \\ &= \frac{2\sigma\phi_0}{m_\phi}\rho_k, \end{aligned} \tag{9}$$

[2] Analyzing in the narrow resonance region, we can regard that the amplitude of inflaton field is approximately constant.

where (7) is used in the second line.

$\dot{\rho}_\chi^k$ can be identified as the production rate of the χ-particle with momentum k due to parametric resonant decay of inflaton field ϕ. Since the production rate is proportional to the energy density of produced χ-particles (hence its occupation numbers in each k mode), we can see that this is an induced effect. That is, if many particles are created in the particular phase space, further production will be "induced".

Now we will find and use the explicit form of the solution for (1) to obtain the previous result. Hamiltonian is a diagonalized operator for the free field, but in the presence of interaction, it is not diagonalized in terms of a and $a^\dagger$. It can be diagonalized at any instant of time by means of Bogolyubov transformations, which give the transition from the free field creation and annihilation operator, $a^\dagger$ and a, to time dependent operators, $b^\dagger(t)$ and $b(t)$ (which preserve the commutation relation),

$$\begin{aligned} b(t) &= \alpha(t)a + \beta^*(t)a^\dagger, \\ b^\dagger(t) &= \beta(t)a + \alpha^*(t)a^\dagger, \end{aligned} \tag{10}$$

and $\alpha(t)$ and $\beta(t)$ can be written as [9]

$$\begin{aligned} \alpha &= \frac{\mathrm{e}^{i\int \Omega_k dt}}{\sqrt{2\Omega_k}}(\Omega_k\chi_- + i\dot{\chi}_-), \\ \beta &= \frac{\mathrm{e}^{-i\int \Omega_k dt}}{\sqrt{2\Omega_k}}(\Omega_k\chi_- - i\dot{\chi}_-), \end{aligned} \tag{11}$$

where $\Omega_k^2 = \omega_k^2 + 2\sigma\phi$, and the initial conditions for α, β are

$$|\alpha(0)| = 1, \qquad \beta(0) = 0. \tag{12}$$

Then the form of the solution can be written as, to the lowest order [9],

$$\chi_- = \frac{1}{\sqrt{2\omega_k}}[X\cos(\omega_k t) - Y\sin(\omega_k t)], \tag{13}$$

where X, Y are complex weakly time-dependent functions. From Eqs.(11) and (12) the initial conditions for χ become

$$i\dot{\chi}_-(0) = \omega_k\chi_-(0), \quad |\chi_-(0)| = \frac{1}{\sqrt{2\omega_k}}, \tag{14}$$

which lead to the initial values for X and Y:

$$X(0) = \mathrm{e}^{i\delta}, \quad Y(0) = i\,\mathrm{e}^{i\delta}, \tag{15}$$

where δ is some phase which is not important and is set to zero below.

We are only interested in the case where the resonance condition holds:

$$\omega_k \approx \frac{m_\phi}{2}. \tag{16}$$

Therefore, the approximate solutions for X and Y are given by

$$\begin{aligned} X &\simeq \frac{1+i}{2}\,\mathrm{e}^{\mu t} + \frac{1-i}{2}\,\mathrm{e}^{-\mu t}, \\ Y &\simeq \frac{1+i}{2}\,\mathrm{e}^{\mu t} - \frac{1-i}{2}\,\mathrm{e}^{-\mu t}, \end{aligned} \tag{17}$$

where $\mu = m_\phi^{-1}\sqrt{\sigma^2\phi_0^2 - \Delta^2}$, and $\Delta = \omega_k^2 - m_\phi^2/4$. Therefore we obtain the squares of the amplitude of χ_- and $\dot\chi_-$ in the form

$$|\chi_-|^2 = \frac{1}{2\omega_k}[\cosh(2\mu t) - \sinh(2\mu t)\sin(2\omega_k t)], \tag{18}$$

$$|\dot\chi_-|^2 = \frac{1}{2\omega_k}\omega_k^2[\cosh(2\mu t) + \sinh(2\mu t)\sin(2\omega_k t)], \tag{19}$$

Taking the average over one period of oscillations of inflaton field ϕ and using Eqs. (18) and (19), we obtain the averaged ρ_χ^k and $\dot\rho_\chi^k$:

$$\langle\rho_\chi^k\rangle \simeq \frac{\omega_k}{2}\cosh(2\lambda t) \simeq \frac{m_\phi}{4}\cosh(2\lambda t) \tag{20}$$

$$\langle\dot\rho_\chi^k\rangle \simeq \sigma\phi_0\sinh(2\mu t)\langle\sin(m_\phi t)\sin(2\omega_k t)\rangle = \frac{1}{2}\sigma\phi_0\sinh(2\mu t), \tag{21}$$

where the non-vanishing value can be achieved only when the resonance condition (16) holds. Therefore, the following relation is found to be satisfied:

$$\dot\rho_\chi^k = \frac{2\sigma\phi_0}{m_\phi}\tanh(2\mu t)\rho_\chi^k \simeq \frac{2\sigma\phi_0}{m_\phi}\rho_\chi^k \qquad (\text{as}\quad \mu t : \text{large}). \tag{22}$$

Here we get the same result as (9).

The total production rate is obtained by summing up all the mode:

$$\begin{aligned} \dot\rho_\chi &= \int \frac{\omega_k^2}{2\pi^2} d\omega_k \rho_\chi^k \\ &\simeq \int \frac{\omega_k^2}{2\pi^2} d\omega_k \frac{1}{2}\sigma\phi_0 \sinh(2\int^{\omega_k}\mu\left|\frac{dt}{d\omega'_k}\right| d\omega'_k) \\ &\simeq \frac{H}{2\pi^2}\left(\frac{m_\phi}{2}\right)^4 \frac{1}{2}\cosh\left(\frac{2\pi\sigma^2\phi_0^2}{Hm_\phi^3}\right), \end{aligned} \tag{23}$$

where μ is regarded as adiabatically changing variable in the resonance band and the integration is taken over only inside the resonance band in the second line and, in the third, we use the resonance condition ($\omega_k \approx m_\phi/2$) and H is the Hubble's constant. Using the relation $\dot{\rho} = \Gamma_\chi^{(res)}\rho_\phi$ (energy conservation), we can calculate the resonant decay rate given by

$$\begin{aligned}\Gamma_\chi^{(res)} &\simeq \frac{H}{2\pi^2\rho_\phi}\left(\frac{m_\phi}{2}\right)^4 \frac{1}{2}\cosh\left(\frac{2\pi\sigma^2\phi_0^2}{Hm_\phi^3}\right) \\ &\simeq \frac{Hm_\phi^2}{64\pi^2\phi_0^2}\exp\left(\frac{2\pi\sigma^2\phi_0^2}{Hm_\phi^3}\right). \qquad (24)\end{aligned}$$

This is identical to the result of ref. [9] (when $\int \mu dt$ is large).

As mentioned before, we can obtain the similar results for other forms of interaction between the inflaton and the χ-particle or for $\frac{\lambda}{4}\phi^4$ potential. [3] Here, we only show the results corresponding to (22). For the interaction $\mathcal{L}_{int} = -g\phi^2\chi^2$, we get

$$\dot{\rho}_\chi^k \simeq \frac{g\phi_0^2}{2m_\phi}\rho_\chi^k, \qquad (25)$$

and for the theory $V = \frac{\lambda}{4}\phi^4$, we obtain

$$\begin{aligned}\dot{\rho}_\chi^k &\simeq \frac{2\sigma}{c\sqrt{\lambda}}\rho_\chi^k, \qquad \text{for} \quad \mathcal{L}_{int} = -\sigma\phi\chi^2, \qquad (26) \\ &\simeq \frac{g\phi_0}{2c\sqrt{\lambda}}\rho_\chi^k, \qquad \text{for} \quad \mathcal{L}_{int} = -g\phi^2\chi^2, \qquad (27)\end{aligned}$$

where $c \sim \mathcal{O}(1)$.

3 Restriction due to dissipative effects

As shown in the previous section, the parametric resonant decay of inflaton field is an induced effect of produced bosonic particles. If there are large occupation numbers of produced bosons, further creation proportional to their occupation numbers will occur. The crucial point is that a lot of bosons can occupy in the same phase space.

On the other hand, if particles in particular phase space decay into other particles or collide with each other, occupation numbers of bosons in that particular phase space cannot grow very fast. As a result, the resonant decay will be much suppressed.

[3] The behavior of inflaton field can be approximated with a sinusoidal function since its amplitude is small in the narrow resonance region.

In other words, the resonance band becomes narrower in the presence of decay or scattering processes.

We begin with the mode equation for the χ field which is produced by the decay of inflaton field ϕ. One convenient way to take into account the decay of the field χ is to introduce a dumping term in the mode equation. [4] Then the equation becomes a type of equation for damping oscillator with an external oscillating force for the theory $V = \frac{1}{2}m_\phi^2\phi^2$ and $\mathcal{L}_{int} = -\sigma\phi\chi^2$ (we can obtain the similar results for different types of effective potential $V(\phi)$ or interaction Lagrangian $\mathcal{L}_{int}$),

$$\ddot{\chi}_k + \Gamma_d\dot{\chi}_k + [\omega_k^2 + 2\sigma\phi_0\sin(m_\phi t)]\chi_k = 0, \tag{28}$$

where Γ_d is the decay rate for the χ field. Then we obtain the solution χ_k for Eq.(28) in the following form:

$$|\chi_-|^2 = \frac{\mathrm{e}^{-\Gamma_d t}}{2\omega_k}[\cosh(2\mu t) - \sinh(2\mu t)\sin(2\omega_k t)]. \tag{29}$$

The evolution of the energy density of the χ-particle in k mode becomes

$$\begin{aligned} \dot{\rho}_\chi^k &= \frac{d}{dt}\left(\frac{1}{2}|\dot{\chi}_-|^2 + \frac{1}{2}\Omega_k^2|\chi_-|^2\right) \\ &= \frac{1}{2}\frac{d(\Omega_k^2)}{dt}|\chi_-|^2 - \Gamma_d|\chi_-|^2 \\ &\simeq \frac{2\sigma\phi_0}{m_\phi}\rho_\chi^k - \Gamma_d\rho_\chi^k, \end{aligned} \tag{30}$$

where we use Eq.(28) in the second line and, in the third, take the average over one period of oscillations of inflaton field ϕ. The first term is the usual resonant production from decay of inflaton field ϕ and the second is the energy decrease due to the decay of the χ-particle. Therefore, the energy density of the χ-particle in the mode k is reduced by the factor $\mathrm{e}^{-\Gamma_d t}$ compared with the case of no dissipation, and hence the considerable decrease of the resonant decay rate of inflaton field.

Summing over mode k, we obtain the total resonant production of energy density of the χ-particle:

$$\dot{\rho}_\chi = \int \frac{\omega_k^2}{2\pi^2}d\omega_k\frac{1}{2}\frac{d(\Omega_k^2)}{dt}|\chi_-|^2$$

[4]To include the dissipative process in the mode equation is not so simple as given in this paper. The precise treatment of dissipation is very complicated subject and beyond the scope of the present paper. However, we believe that the simple argument given here is reasonable in the physical point of view.

$$
\begin{aligned}
&\simeq \int \frac{\omega_k^2}{2\pi^2} d\omega_k \frac{1}{4}\sigma\phi_0 \exp(\int^{\omega_k} (2\mu - \Gamma_d) \left|\frac{dt}{d\omega_k'}\right| d\omega_k') \\
&\simeq \frac{H}{2\pi^2}\left(\frac{m_\phi}{2}\right)^4 \frac{1}{4} \exp\left[\frac{4\sigma^2\phi_0^2}{Hm_\phi^3}\left(\sin^{-1}\sqrt{1-\left(\frac{m_\phi\Gamma_d}{2\sigma\phi_0}\right)^2} - \frac{m_\phi\Gamma_d}{2\sigma\phi_0}\sqrt{1-\left(\frac{m_\phi\Gamma_d}{2\sigma\phi_0}\right)^2}\right)\right],
\end{aligned}
\tag{31}
$$

where, in the third line, we take into account that the resonance band width gets narrower due to dissipation: $\Delta = \sigma\phi_0 \to \sigma\phi_0\sqrt{1-\left(\frac{m_\phi\Gamma_d}{2\sigma\phi_0}\right)^2}$. If $\Gamma_d < \sigma\phi_0/m_\phi (\simeq \mu)$, the production rate becomes approximately,

$$
\dot{\rho}_\chi \simeq \frac{H}{2\pi^2}\left(\frac{m_\phi}{2}\right)^4 \frac{1}{4} \exp\left[\frac{2\pi\sigma^2\phi_0^2}{Hm_\phi^3}\left(1 - \frac{2m_\phi\Gamma_d}{\pi\sigma\phi_0}\right)\right]. \tag{32}
$$

Now we can see that the resonant production of χ-particles is reduced due to dissipative effects, and hence the reduction of resonant decay rate of inflaton field ϕ.

Simpler explanation can be given as follows. The naive integration in the exponent in Eq.(31) becomes

$$
\begin{aligned}
\int (2\mu - \Gamma_d) dt &= \int 2\mu dt - \int \Gamma_d dt \\
&\simeq \int 2\mu dt - \Gamma_d \tau_{res},
\end{aligned}
\tag{33}
$$

where we regard Γ_d as constant, and τ_{res} is the duration of the parametric resonance (the time while staying in the resonance band). If $\tau_{res} > \Gamma_d^{-1}$, i.e., the lifetime of the χ-particle is shorter than the resonant duration, the particle does not stay in the particular phase space long enough that the occupation number in that particular mode cannot grow so fast, which results in less resonant production.

4 Conclusion

We investigate the parametric resonant decay of inflaton field ϕ and the dissipative effects of particles produced after the chaotic inflation. The equation for the particles produced can be described by Mathieu type equation, which has instability solution in some regions of parameters (instability bands). If the frequencies stay in the instability bands long enough, the solution will explosively grow, so that the number of created particles becomes exponentially large. Therefore the previous authors [8, 9, 10, 11, 12] regarded that the parametric resonant decay is important.

In this article, we considered this phenomena from another point of view. The resonant decay is an induced effect of bosonic particles. If there are large occupation numbers of produced bosons, further creation proportional to their numbers will occur. A lot of bosons can occupy in the same phase space while one fermion can occupy in the particular phase space. This is the reason that the inflaton field does not decay explosively into fermions (Pauli's exclusion principle) [14, 8, 9, 10].

Then, in the case of decaying into bosons, if the dissipative effects prevent the occupation numbers of bosons from growing very fast, the resonant decay will be much suppressed. The dissipative processes involve collisions between particles created, and their decay or annihilation into other lighter particles.

The simplest way to look for this effect is compare the duration of resonance (the time that resonance frequency stays in the instability band) with the lifetime of produced particles. If the resonant duration is shorter than the particle lifetime, resonance occurs considerably. In the opposite case, parametric resonance should be extremely suppressed. We believe that this criterion is general.

The dissipative effects are taken into account phenomenologically by $\Gamma_d \dot{\chi}_k$ term in the equation for the mode k of the field χ, although it is difficult to obtain the term $\Gamma_d \dot{\chi}_k$ from the first principle. Besides, decay rates of particles depend on its energy which is redshifted by the expansion of the Universe, so that the investigation is much more complicated. However, we expect that the result will remain the same qualitatively.

There are some cosmological implication from this effects. First, those particles that are created by the decay of inflaton field should weakly interact with themselves or with other particles. Otherwise, the dissipative processes become effective so as to reduce the resonant decay of inflaton field considerably. Second, the reheating temperature will not be so high because thermalization takes for a long time.

Finally we must mention in closing that our simple analytical investigation in this article for both parametric resonance and dissipative effects will be limited to only in the narrow resonance region where the effects of back-reaction to the amplitude of inflaton filed is rather small. There are some papers studying broad resonance, which is rather complicated [8, 10, 11, 15]. However, we expect that the basic physical process for broad resonance is the same as narrow one. Therefore, an induced nature of parametric resonant decay will also hold qualitatively in the broad resonace region so that the dissipation tends to suppress the resonant decay in the same way.

(It is model-dependent whether or not this suppresion becomes important, and we expect that it will be less efficient in the broad resonance region, since the resonance duration will be much shorter than the lifetime of created particles.)

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